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INVENTORY AVERAGE COSTS: NON-UNIT ORDER SIZES AND RANDOM LEAD TIMES

TECHNICAL REPORT

BY

RICHARD URBACH

MARCH 1977

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Paper addresses inventory system with non-unit order sizes and variable lead times under a continuous review (S-s) policy. Under the assumption of at most one order outstanding, some quite general results are obtained. An example with a specific form of demand and lead time distribution is then analyzed and computation formulas developed. Finally, a heuristic approach is proposed for case when there is likelihood of more than one order outstanding.

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INTRODUCTION

For some time there has been considerable concern with average cost computations in inventory systems with non-unit order sizes and variable lead times when following a stationary (S,s) policy. In this paper the derivation of the stationary distribution of the inventory position was motivated to a large extent by Izzet Sahin's work reported in [2]; however, in that paper much time was spent on deriving finite horizon equations and then taking limits. We circumvent the latter by applying the well known age distribution for renewal processes. The work also generalizes Silver's [3] results on the stationary distribution of inventory position.

In the first chapter we derive the stationary distribution of the inventory position and, under the assumption of at most one order outstanding, derive the joint stationary distribution of the inventory position and the number of orders outstanding. The latter distribution permits us to compute approximately the average cost for given demand and lead time distributions.

In the second chapter we analyze an example with a specific form of demand and lead time distribution; developing recursion formulae for fast computation of the joint stationary distribution of inventory position and number of orders outstanding. Also given is a listing of a computer program which implements these recursion formulae and finds the minimum cost value of s for a fixed S.

The third chapter takes a heuristic approach and gives an average cost equation based on this author's intuitive notions on the ergodic properties of some of the processes involved, plus some outright approximations. No claims to rigor are made here, but then of course the analysis is short and sweet and its adequacy can be tested via simulation.

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CHAPTER I

RIGOROUS ANALYSIS

Consider a continuous review inventory system. Requisitions for inventory items occur at random points in time and the size of each requisition is also random. The ordering policy is (S,s) and the lead times are random.

If $I_p(t)$ is the inventory position (on hand plus on order) at time t then the times at which orders are placed are renewal points for the $I_p(t)$ process. Define

$$N(t) = \text{number of orders placed by time } t$$

$$T_n = \text{time between the } n^{th} \text{ and } n\text{-}1^{st} \text{ order}$$

and $$V(t) = t - \sum_{n=1}^{N(t)} T_n$$

V(t) is the time since the last order was placed and is sometimes called the age at time t.

If we let an asterisk denote the stationary version of a process then it is well known that (see [3] pp. 45)

$$F_{V^*}(y) = \frac{1}{ET^*} \int_0^y [1-F_{T^*}(u)]du$$

so if $\Delta = S-s$ then for $s < x \leq S$ and $D(\cdot)$ cumulative demand

$$P[I_P^* = X] = \int_0^\infty P[I_P^* = X \mid V^* = y] \, f_{V^*}(y)dy$$

$$= \frac{1}{ET^*} \int_0^\infty P[D(y) = S - X \mid V^* = y] \, [1-F_{T^*}(y)] \, dy$$

$$= \frac{1}{ET^*} \int_0^\infty P[D(y) = S - X \mid D(y) < \Delta][1-F_{T^*}(y)]dy$$

$$= \frac{1}{ET^*} \int_0^\infty P[D(y) = S - X]dy \qquad (1)$$

since $V^* = y \iff D(y) < \Delta$, and a renewal at $-y$

Moreover $\int_0^\infty P[D(y) = S - X]dy$

$$= E \int_0^\infty I_{[D(y) = S - X]} dy \tag{2}$$

$= E$[amount of time cumulative demand was at S-X]

$$= \frac{P[\text{cumulative demand was ever at } S-X]}{\lambda}$$

where $\lambda \equiv 1/E$ (time between requisitions).

Let

$B(X)$ = CDF of the requisition size and $B_n(X)$ its n-fold convolution ($B_o(X) = 1$)

$A(X)$ = CDF of cumulative requisitions and $A_n(X)$ its n-fold convolution ($A_o(X) = 1$)

$$H(X) = \sum_{n=1}^{\infty} B_n(X) \quad ; \quad H(o) = o$$

It is well known that H is the renewal function (expected cumulative renewals) for the process with inter-renewal distribution B, i.e., the time-between-renewals distribution is just the distribution of requisition size and H(X) is the expected number of requisitions in a total demand of X.

$$h(X) \equiv H(X) - H(X-1) = P[\text{Renewal at } X] \qquad x > o$$

and a renewal at X for this process is the same as the event that cumulative demand is ever at X, hence from (2)

$$h(X) = \lambda \int_0^\infty P[D(y) = X]\, dy \tag{3}$$

Since the T_n are iid T^* and T_n have the same distribution and it is clear that

$$T > y \iff D(y) < \Delta$$

Hence

$$ET = \int_o^\infty P[T > y]dy = \int_o^\infty P[D(y) < \Delta]\, dy$$

$$= \sum_{X=1}^{\Delta-1} \int_o^\infty P[D(y) = X]\, dy \quad + \int_o^\infty P[D(y) = o]\, dy$$

$$= \sum_{X=1}^{\Delta-1} \frac{h(X)}{\lambda} \quad + \int_o^\infty P[D(y) = o]\, dy$$

$$= 1/\lambda\, [H(\Delta-1) - H(o)] + 1/\lambda$$

$$= \frac{1}{\lambda}\, [H(\Delta-1) + 1] \qquad (4)$$

Use the natural extension of (3) to get $h(o) = \lambda \int_o^\infty P[D(y) = o]\, dy = 1$ and equations 1-4 to get

$$P[I^* = X] = \frac{h(S-X)}{1 + H(\Delta-1)} \qquad s < X \leq S$$

Let

L = lead time of an order

$Z(t)$ = number of orders outstanding at time t

Suppose the probability that the lead time demand is greater than Δ-1 is negligible. Then we can safely assume that there is never more than one order outstanding, i.e., $Z(t)$ is o or 1 almost surely. It then follows that

$$P[I_p^* = X,\ Z^* = o] = \frac{1}{ET} \int_o^\infty P[D(y) = S - X]\, P[L \leq y]\, dy$$

$$= \frac{\lambda}{1+H(\Delta-1)} \int_o^\infty P[D(y) = S - X]\, P[L \leq y]dy$$

and

$$P[I_p^* = X,\ Z^* = 1] = \frac{h(S-X)}{1+H(\Delta-1)} - P[I_p^* = X,\ Z^* = o]$$

since $P[I_p^* = X] = P[I_p^* = X,\ Z^* = 1] + P[I_p^* = X,\ Z^* = o]$

these last equations can be used to compute approximately

$$E\ [\text{Average backorder and holding cost}]$$

$$\doteq\ HC \cdot E(I_p^* - Z^*\Delta)^+ + HB \cdot E(I_p^* - Z^*\Delta)^-$$

where we have approximated the outstanding order size by Δ (it is in general larger) and

HC = holding cost per item per unit time

HB = backorder cost per item per unit time

CHAPTER II

EXAMPLE

If the time between requisitions is exponential with parameter λ and the requisition sizes are geometric with parameter p, i.e.,

$$p(x) = pq^{x-1} \qquad x = 1,2,\ldots \quad ; \quad q = 1 - p$$

then

$$P[D(y) = x] = \begin{cases} e^{-\lambda y} & x = o \\ e^{-\lambda y} q^x \sum_{j=1}^{x} \binom{x-1}{j-1} \frac{1}{j!} \left(\frac{\lambda y p}{q}\right)^j & x > o \end{cases}$$

$$B_n(x) = \begin{cases} o & n > x \\ \sum_{k=n}^{x} \binom{k-1}{n-1} p^n q^{k-n} & n \le x \end{cases}$$

$$H(x) = \sum_{n=1}^{x} B_n(x) = \sum_{n=1}^{x} \sum_{k=n}^{x} \binom{k-1}{n-1} p^n q^{k-n}$$

$$= \sum_{k=1}^{x} \sum_{n=1}^{k} \binom{k-1}{n-1} p^n q^{k-n}$$

$$= \sum_{k=1}^{x} p \sum_{n=o}^{k-1} \binom{k-1}{n} p^n q^{k-1-n}$$

$$= \sum_{k=1}^{x} p = px$$

so that

$$ET = 1/\lambda \; (1 + p(\Delta-1))$$

$$h(x) = \begin{cases} 1 & x = o \\ p & x > o \end{cases}$$

and

$$\int_z^\infty P[D(y) = x]dy = \begin{cases} e^{-\lambda z}/\lambda & x = o \\ (q^x e^{-\lambda z}/\lambda) \sum_{j=1}^{x} a_j(x,z) & x > o \end{cases}$$

where

$$a_j(x,z) = \binom{x-1}{j-1} (p/q)^j \sum_{k=o}^{j} (\lambda z)^k k!$$

Suppose the support of L is a finite lattice, i.e., L assumes only a finite number of values. Let

$$P[L = g_i] = y_i \qquad i = 1,2,\ldots m$$

Then we can write

$$P[I_p^* = x,\ Z^* = o]$$

$$= \int_o^\infty P[D(y) = x]\ P[L \leq y]dy$$

$$= y_1 \int_{g_1}^{g_2} P[D(y) = x]dy + y_1+y_2) \int_{g_2}^{g_3} P[D(y)=x]dy$$

$$+ \ldots + (y_1 + \ldots + y_m) \int_{g_m}^\infty P[D(y) = x]dy$$

$$= y_1 \int_{g_1}^\infty P[D(y) = x]dy + y_2 \int_{g_2}^\infty P[D(y) = x]dy$$

$$+ \ldots + y_m \int_{g_m}^\infty P[D(y) = x]dy$$

$$= \begin{cases} \sum_{i=1}^{m} y_i e^{-\lambda g_i/\lambda} & x = o \\ \sum_{i=1}^{m} (q^x e^{-\lambda g_i/\lambda}) \sum_{j=1}^{x} a_j(x,g_i) & x > o \end{cases}$$

For computations let

$$b_j = \sum_{k=o}^{j} (\lambda z)^k/k!$$

and

$$c_j = (\lambda z)^j/j!$$

so that

$$a_{j+1} = a_j \, [1 + \lambda z c_j/(j+1)b_j][\tfrac{x-j}{j}] \, (p/q)$$

This recursion affords a straight forward and fast algorithm for computing $P[I_p^* = x, Z^* = o]$.

Remark: For this example empirical investigations show that the average cost is a convex function of s.

Appendix A contains a listing of a computer program which implements these recursion formulae and finds the minimum cost value of s for a given order-up-to point, backorder cost, holding cost, requisition rate, and average requisition size. To use it

CALL OPTRPT (S,IS, BC, HC, CO, L, P)

INPUT:
BC = unit backorder cost
HC = unit holding cost
S = order up to point
L = requisition rate
P = recipricol of the average requisition size

OUTPUT:
IS = optimal reorder point
CO = optimal cost

CHAPTER III

HEURISTIC ANALYSIS

As an alternative approach consider points in time at which orders arrive. The average cost incurred between two of these points is given by (see figure below)

$$C_M(N) = \frac{C_H \frac{Lo}{M+1} \sum_{n=o}^{M} (N - nEr)^+ + C_B \frac{Lo}{M+1} \sum_{n=o}^{M} (N - nEr)^-}{Lo}$$

$$= \left[C_H \sum_{n=o}^{M} (N-nEr)^+ + C_B \sum_{n=o}^{M} (N-nEr)^-\right] \frac{1}{M+1}$$

where Lo = time between the arrival of the two orders

M = number of requisitions in Lo

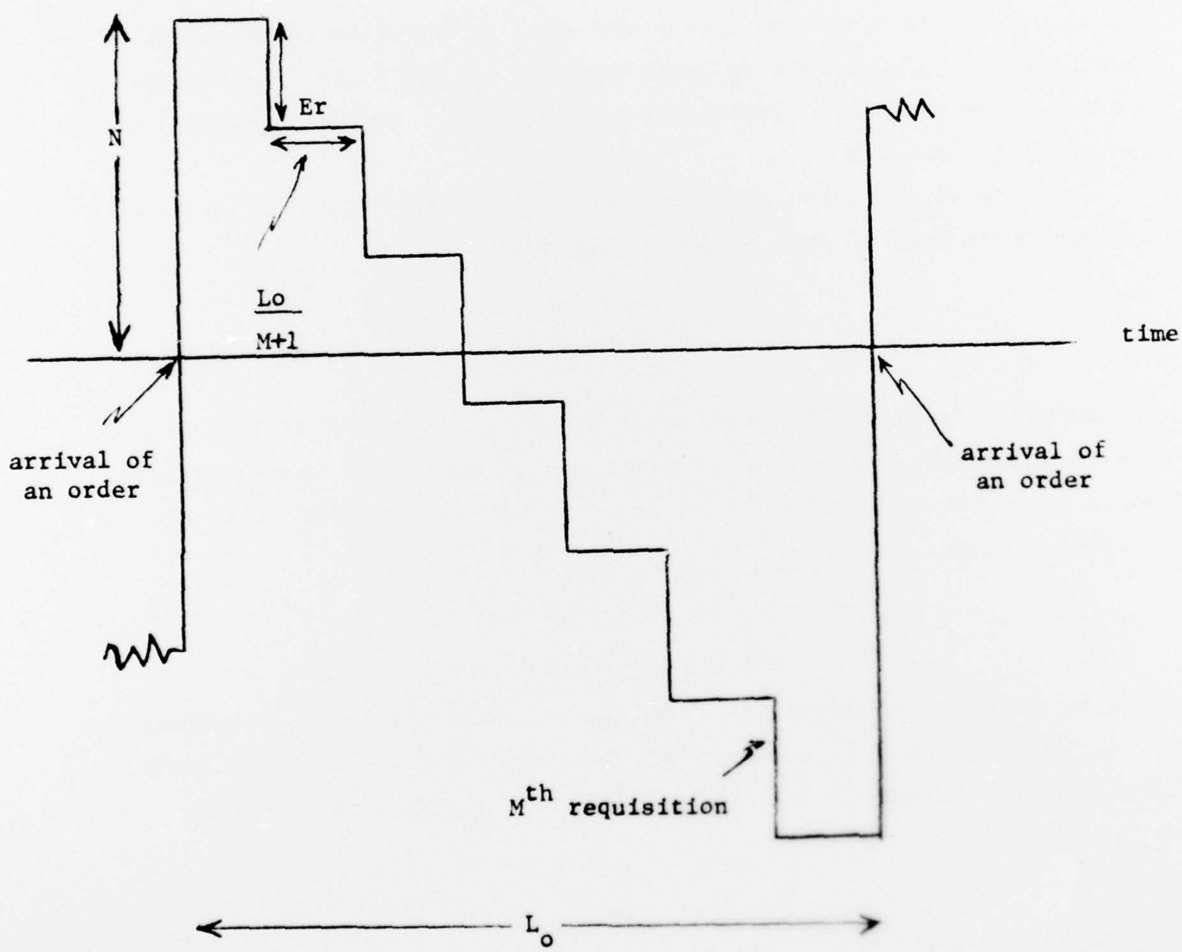
Er = expected requisition size

N = net inventory just after first order arrives

C_B = unit backorder cost

C_H = unit holding cost

FIGURE



N
Er
Lo
M+1
time
arrival of
an order
arrival of
an order
M th requisition
L o

Let ET be the expected time between the placement of orders. Since orders are being placed at a rate 1/ET, so must the orders be arriving at the same rate.

So lets be bold and replace ELo with ET to get EM = λELo = λET. It looks like the necessary ergodic properties should hold so that we can write

$$EC_M(N) = [C_H E \sum_{n=o}^{M} (N - nEr)^+ + C_B \; E \sum_{n=o}^{M} (N - nEr)^-] \frac{1}{\lambda ET+1}$$

Assuming there is always more than one order outstanding the order of smoothing in the numerator is arbitrary since M and N are independent. However, M depends on Lo inspite of the notation. Replace Lo with T to get the distribution of M.

For the distribution of N (net inventory at the time that an order arrives) note that if orders cannot pass then

$$N = S - D(L)$$

Because in this case all orders outstanding at t-L are in at time t, all orders placed after time t-L are still out at time t and the inventory position at t-L is S since t-L is an order point if t is the time the order arrived. Hence

$$P[N = x] = P[S - D(L) = x]$$

$$= P[D(L) = S - x]$$

If L is on a finite lattice then the above is not difficult to compute if we have the distribution of D(y); for example, the cumulative demand distribution derived in the analytic approach of the first chapter.

APPENDIX A

```
***********************************************************************
SUBROUTINE OPTIMP(S,IS,EC,HC,CO,L,P)
*COMPUTES OPTIMAL REORDER POINT
INTEGER S,SM1
REAL L
COMMON P,L
SM1=S-1
CALL COST(S,0,EC,HC,C0)
DO 10 I=1,SM1
CALL COST(S,I,EC,HC,C1)
IS=I-1
IF(C1.GT.C0) GO TO 20
C0=C1
10 CONTINUE
20 RETURN
END
***********************************************************************
SUBROUTINE COST(S,SL,EC,HC,C0)
INTEGER S,SL,S1,SM1
REAL L
DIMENSION R(2,30)
COMMON P,L
S1=SL+1; I=S-SL; SM1=S-1
E1=(1.+P*(I-1.))/L
V=P/(L*E1)
DO 10 J=S1,SM1
K=S-J
R(1,J+1)=PN(K)/E1
R(2,J+1)=V-R(1,J+1)
10 CONTINUE
R(1,1)=PN(0)/E1
R(2,1)=(V/P)-R(1,1)
*CALL CHECK(R,S1,S)
CALL TOLLAR(R,S,SL,EC,HC,C0)
RETURN
END
***********************************************************************
FUNCTION PN(K)
*COMPUTES P(N*=K,Q*=N)
DIMENSION X(6),Y(6)
DATA X/15.,30.,45.,60.,75.,90./
DATA Y/.1,.4,.3,.15,.04,.01/
SUM=0.0
DO 10 I=1,6
10 SUM=SUM+X[illegible](X(I),K)*Y(I)
PN=SUM
RETURN
END
***********************************************************************
```

```
************************************************************
FUNCTION [illegible]([illegible],[illegible])
*COMPUTES [illegible] FUNCTION, F(Y)=[illegible]
REAL L
COMMON [illegible]
[illegible]
[illegible]=1.-[illegible]
[illegible]=(([illegible]**[illegible])/[illegible])*EXP(-[illegible])
SUM=1.0
IF([illegible].[illegible].0) GO TO 11
SUM=[illegible]*(1.+[illegible])
[illegible]=1.+[illegible]
IF([illegible].[illegible].1) GO TO 11
DO 10 J=2,N
[illegible]=[illegible]*([illegible]-J+1)/([illegible]-1)
[illegible]=1.+[illegible]*C/(J+[illegible])
[illegible]=[illegible]*[illegible]
SUM=SUM+[illegible]
C=C*[illegible]/[illegible]
[illegible]=[illegible]
10 CONTINUE
11 CONTINUE
[illegible]=[illegible]*SUM
RETURN
END
************************************************************
SUBROUTINE [illegible]([illegible],[illegible],[illegible])
INTEGER S1,S
DIMENSION [illegible](2,50)
SUM=0.0
DO 10 J=1,S
I=S-J+1
TEMP=[illegible](1,I)+[illegible](2,I)
WRITE(1,100) [illegible](1,I),[illegible](2,I),TEMP
SUM=SUM+TEMP
10 CONTINUE
WRITE(1,100) SUM
100 FORMAT(1X,5F10.7)
RETURN
END
************************************************************
SUBROUTINE [illegible]([illegible],[illegible],SL,PC,HC,CO)
*COMPUTES EXPECTED BACKORDER + HOLDING COST
REAL L
INTEGER S,S1,[illegible],I
DIMENSION [illegible]([illegible],50)
S1=SL+1; I=S-SL
SUM=0.0
DO 10 I=S1,S
J=S-I+1
DO 10 JJ=1,[illegible]
J=JJ-1
C=[illegible](JJ,I)*FLOAT(I-[illegible])
SUM=SUM+HC*AMAX1(C,0.)-PC*AMIN1(C,0.)
10 CONTINUE
CO=SUM
RETURN
END
```

REFERENCES

1 E.A. Silver, Chung-Mei Ho, Robert Deemer, "Cost-Minimizing Inventory Control of Items Having a Special Type of Erratic Demand Pattern," Infor., Vol 9, No. 3, November 1971, 198-219.

2 I. Sahin, "On the Stationary Analysis of (S,s) Inventory Systems With Lead Times," Working Paper 76-10, Faculty of Management Sciences, University of Ottawa.

3 S. Ross, Applied Probability Models with Optimization Applications, Holden-Day, San Francisco, 1970.

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